Topic: Product rule, three or more functions

Question: Find the derivative.

$$f(x) = 3x^2e^x \sin x$$

Answer choices:

A $f'(x) = 6xe^x \cos x$

B $f'(x) = 6xe^x \sin x + 6xe^x \cos x + 3x^2e^x \cos x$

C $f'(x) = 6xe^x \sin x + 3x^2e^x \cos x$

D $f'(x) = 6xe^x \sin x + 3x^2e^x \sin x + 3x^2e^x \cos x$

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Solution: D

First, note that $f(x) = 3x^2e^x \sin x$ is the product of three separate functions of our variable x: $3x^2$, e^x and $\sin x$. Therefore, to differentiate $f(x)$, we'll need to use the product rule for three functions:

$$f(x) = g(x) \cdot h(x) \cdot k(x)$$

$$f'(x) = g'(x) \cdot h(x) \cdot k(x) + g(x) \cdot h'(x) \cdot k(x) + g(x) \cdot h(x) \cdot k'(x)$$

In this case, let's let $g(x) = 3x^2$, $h(x) = e^x$ and $k(x) = \sin x$. Then we can separately find the derivatives of each of these three functions:

$$g(x) = 3x^2$$

$$g'(x) = 3(2x) = 6x$$

and

$$h(x) = e^x$$

$$h'(x) = e^x$$

and

$$k(x) = \sin x$$

$$k'(x) = \cos x$$

We can now find $f'(x)$ by substituting these results into our three-function product rule formula:

$$f'(x) = g'(x) \cdot h(x) \cdot k(x) + g(x) \cdot h'(x) \cdot k(x) + g(x) \cdot h(x) \cdot k'(x)$$

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$$f'(x) = 6xe^x \sin x + 3x^2e^x \sin x + 3x^2e^x \cos x$$

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Topic: Product rule, three or more functions

Question: Find the derivative without multiplying out the factors.

$$f(x) = (x+3)\left(2x^2-5\right)\left(-x^3+2\right)$$

Answer choices:

A $f'(x) = -12x^3$

B $f'(x) = \left(2x^2-5\right)\left(-x^3+2\right) + 4x(x+3)\left(-x^3+2\right) - 3x^2(x+3)\left(2x^2-5\right)$

C $f'(x) = -12x^3(x+3) - 3x^2\left(2x^2-5\right) + 4x\left(-x^3+2\right)$

D $f'(x) = (x+3)\left(4x-3x^2\right) + \left(2x^2-5\right)\left(-x^3+2\right)$

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Solution: B

First, note that we could find the derivative by first multiplying out the factors of this function and finding the derivative of each resulting term. However, since the function is currently written as the product of three functions, we can also use the product rule to find its derivative:

$$f(x) = g(x) \cdot h(x) \cdot k(x)$$

$$f'(x) = g'(x) \cdot h(x) \cdot k(x) + g(x) \cdot h'(x) \cdot k(x) + g(x) \cdot h(x) \cdot k'(x)$$

In this case, let's let $g(x) = x + 3$, $h(x) = 2x^2 - 5$ and $k(x) = -x^3 + 2$. Then we can separately find the derivatives of each of these three functions:

$$g(x) = x + 3$$

$$g'(x) = 1 + 0 = 1$$

and

$$h(x) = 2x^2 - 5$$

$$h'(x) = 2(2x) - 0 = 4x$$

and

$$k(x) = -x^3 + 2$$

$$k'(x) = -3x^2 + 0 = -3x^2$$

We can now find $f'(x)$ by substituting these results into our three-function product rule formula:

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$$f'(x) = g'(x) \cdot h(x) \cdot k(x) + g(x) \cdot h'(x) \cdot k(x) + g(x) \cdot h(x) \cdot k'(x)$$

$$f'(x) = (1)\left(2x^2 - 5\right)\left(-x^3 + 2\right) + (x + 3)(4x)\left(-x^3 + 2\right) + (x + 3)\left(2x^2 - 5\right)\left(-3x^2\right)$$

$$f'(x) = \left(2x^2 - 5\right)\left(-x^3 + 2\right) + 4x(x + 3)\left(-x^3 + 2\right) - 3x^2(x + 3)\left(2x^2 - 5\right)$$

Note that if we were to multiply out this result, we should achieve the same result as if we had multiplied out the original function and taken the derivative from there.

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Topic: Product rule, three or more functions

Question: Find the derivative *without* multiplying out the factors.

$$y = \left(3x^5 - 4\right)^3$$

Answer choices:

A $\dfrac{dy}{dx} = 45x^4\left(3x^5 - 4\right)^2$

B $\dfrac{dy}{dx} = \left(15x^4\right)^3$

C $\dfrac{dy}{dx} = 3\left(15x^4\right)^2\left(3x^5 - 4\right)$

D $\dfrac{dy}{dx} = 3\left(15x^4\right)^3$

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Solution: A

To differentiate this function using the product rule, let's first rewrite the function as

$$y = \left(3x^5 - 4\right)\left(3x^5 - 4\right)\left(3x^5 - 4\right)$$

Now we can see that this function is the product of three factors, and so we can use the product rule to find its derivative.

$$y = u \cdot v \cdot w$$

$$\frac{dy}{dx} = \frac{du}{dx} \cdot \ v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}$$

In this case, $u = 3x^5 - 4$, $v = 3x^5 - 4$ and $w = 3x^5 - 4$. Next let's find the derivative of each factor (note that since each factor is identical, their derivatives are identical as well):

$$u = 3x^5 - 4$$

$$\frac{du}{dx} = 3\left(5x^4\right) = 15x^4 = \frac{dv}{dx} = \frac{dw}{dx}$$

We can now find dy/dx by substituting these results into our three-function product rule formula.

$$\frac{dy}{dx} = \frac{du}{dx} \cdot \ v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = 15x^4 \cdot \ \left(3x^5 - 4\right) \cdot \left(3x^5 - 4\right) + \left(3x^5 - 4\right) \cdot 15x^4 \cdot \left(3x^5 - 4\right) + \left(3x^5 - 4\right) \cdot \left(3x^5 - 4\right) \cdot 15x^4$$

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$$\frac{dy}{dx} = 3 \cdot \left[15x^4 \cdot \ \left(3x^5 - 4\right) \cdot \left(3x^5 - 4\right)\right] = 45x^4 \left(3x^5 - 4\right)^2$$

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